Connected Working Spaces: modelling in the digital age

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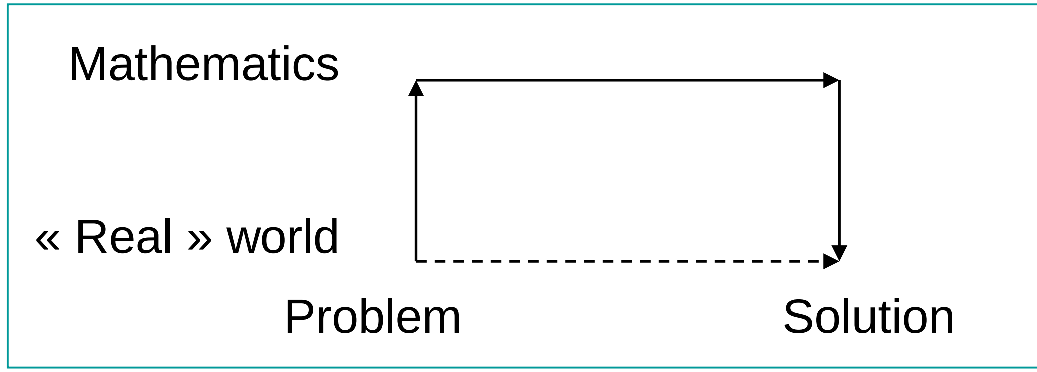
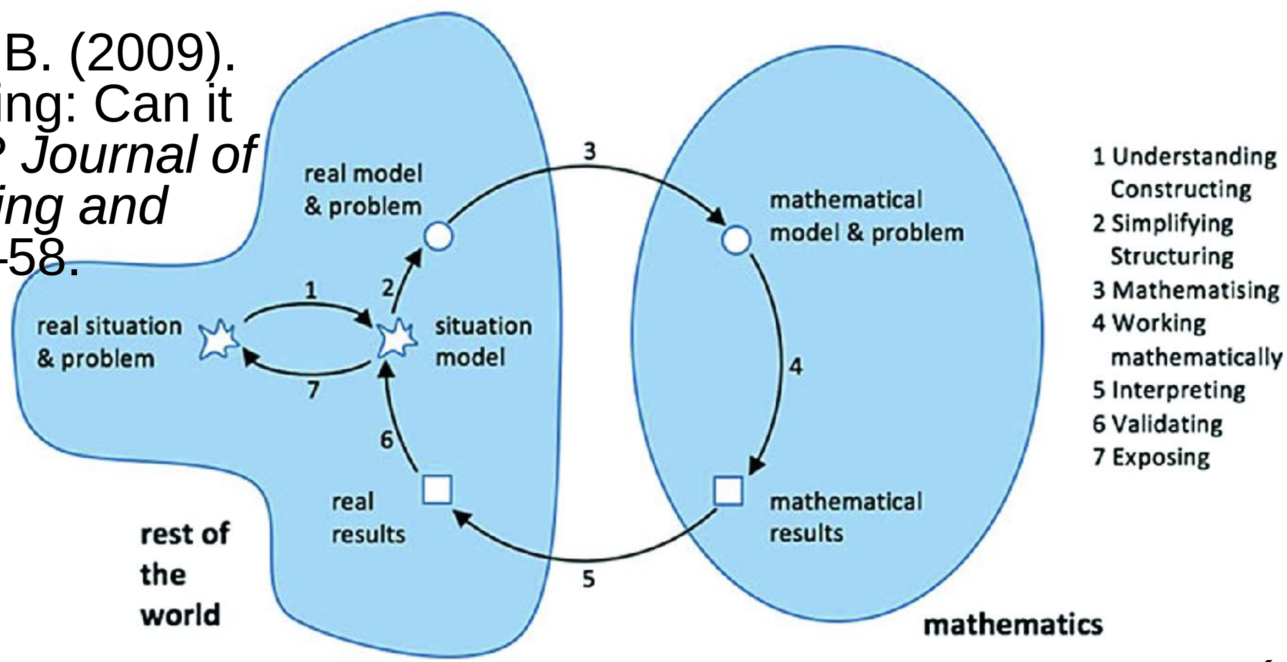
Modelling



- Blum, W., & Ferri, R. B. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, *1*(1), 45–58.



How does validating activity contribute to the modeling process?

Czocher (2018) ESM 99

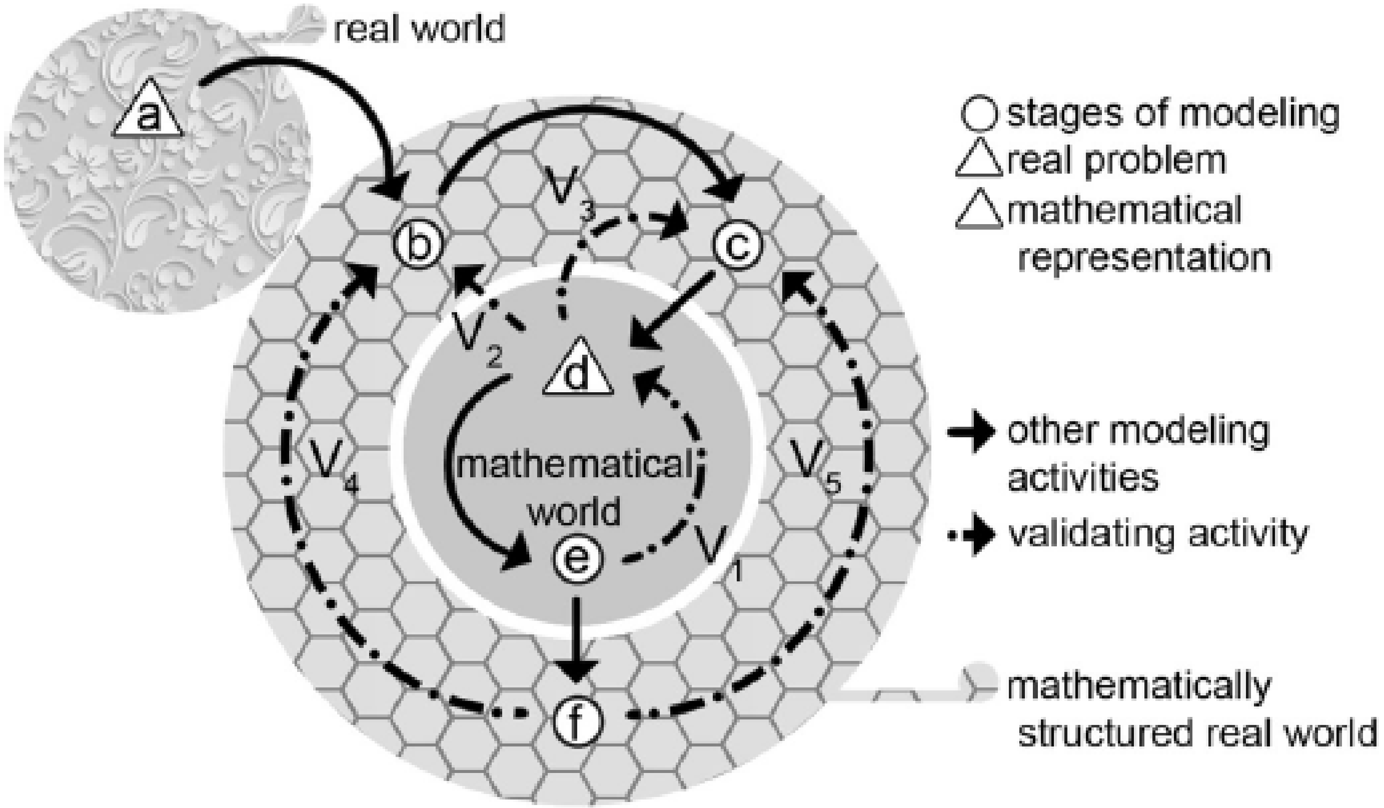


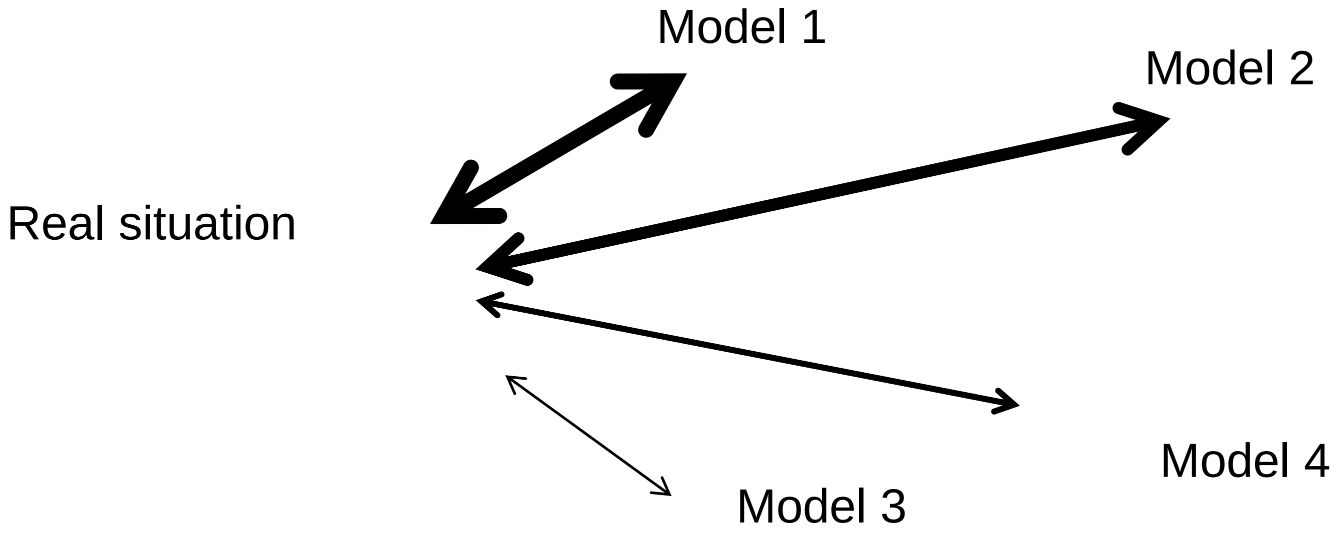
Fig. 5 A modified modeling cycle depicting the typology of validating as the source of idiosyncrasies

« Validation integrates real-world reasoning with mathematical reasoning »

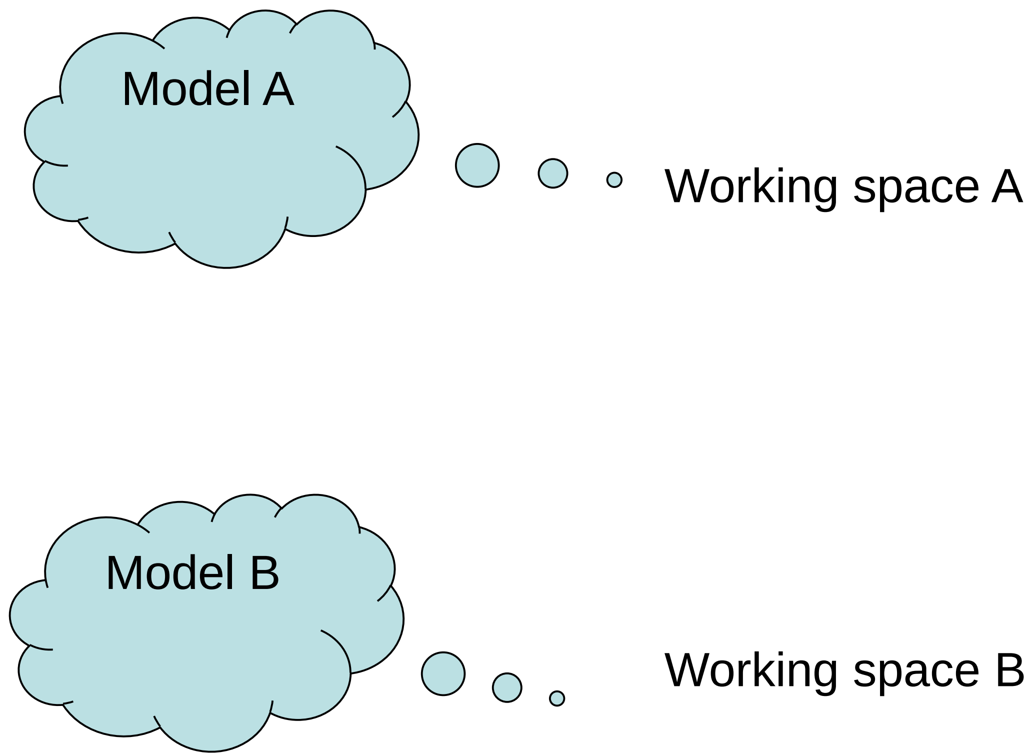
From epistemological studies

- modelling is not merely mathematizing
- for a given reality,
  - there is a plurality of models, allowing
    - operationality (simulation)
    - as well as interpretation (debate)
  - mathematical work
    - is done in close conjunction with work in scientific experimental fields,
    - aims to clarify and simplify models.

Different models of a situation



Working on each model is working in a specific space



A Mathematical Working Space (MWS)

- An abstract space organized to ensure the mathematical work (in an educational context).
- Three dimensions of the work
  - Semiotic : use of symbols, graphics, concrete objects understood as signs,
  - Instrumental: construction using artefacts (geometric figure, program..)
  - Discursive: justification and proof using a theoretical frame of reference (definitions, properties…)

A typical "innovative" situation in French high school

Optimizing the area of a given surface in relationship with a « real life » situation

Phase 1. Make a dynamic geometry figure. Explore and conjecture the optimal figure

Phase 2. Prove the result algebraically, taking a given length for *x* and calculating the area as a function of *x*.

Juxtaposition of two phases

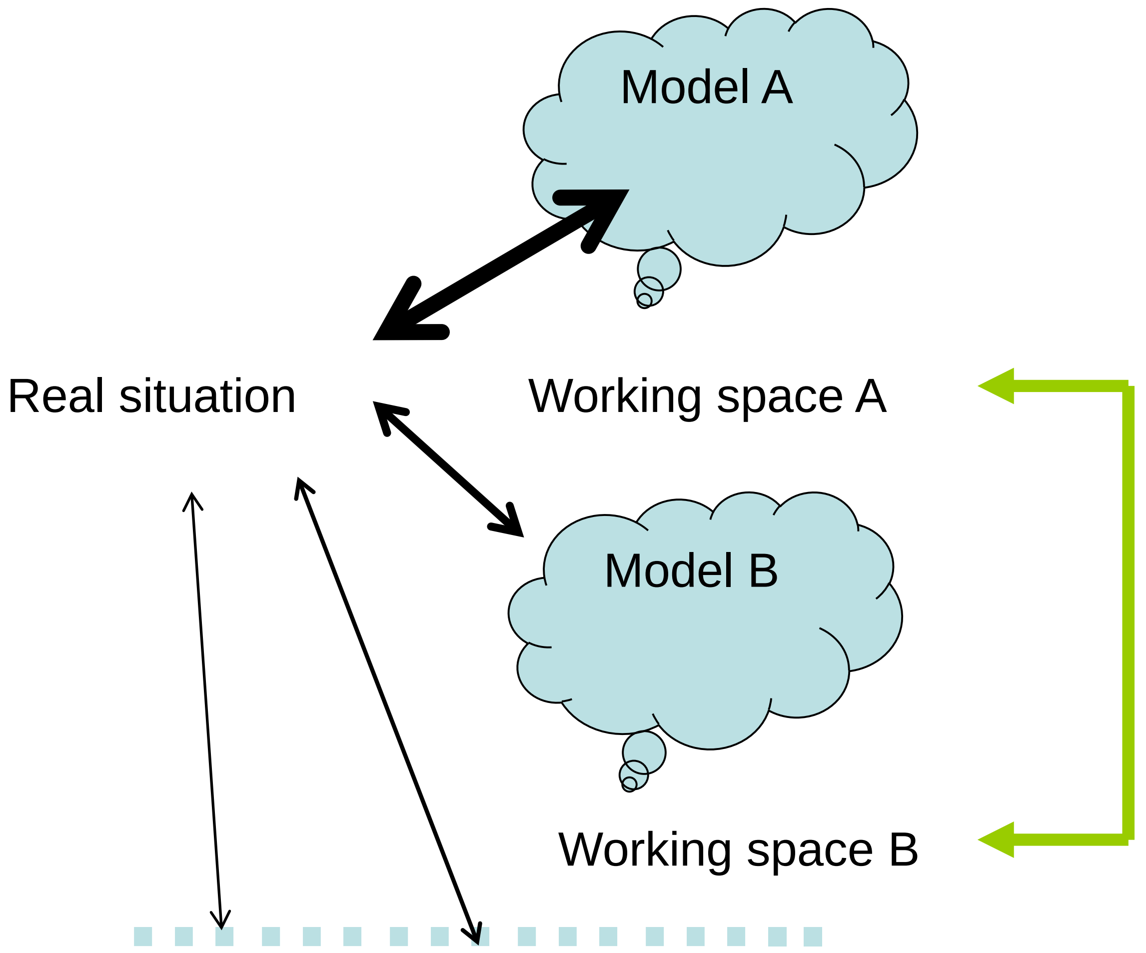
Phase 1 :

- No real working space.
- Motivation.
- Instrumental-Semiotic.

Phase 2:

- Reduction to algebraic calculation working space
- Semiotic-Discursive

Choices and Hypothesis: deeper understanding by connecting working spaces



Careful specification of working spaces

Special classroom organisation to promote connections

1. Groups of experts
2. Groups of discussion

Second Group Work
(discussion)

Task: Find connections between models

First Group Work (experts)
Each group works on a model (A, B, C or D)

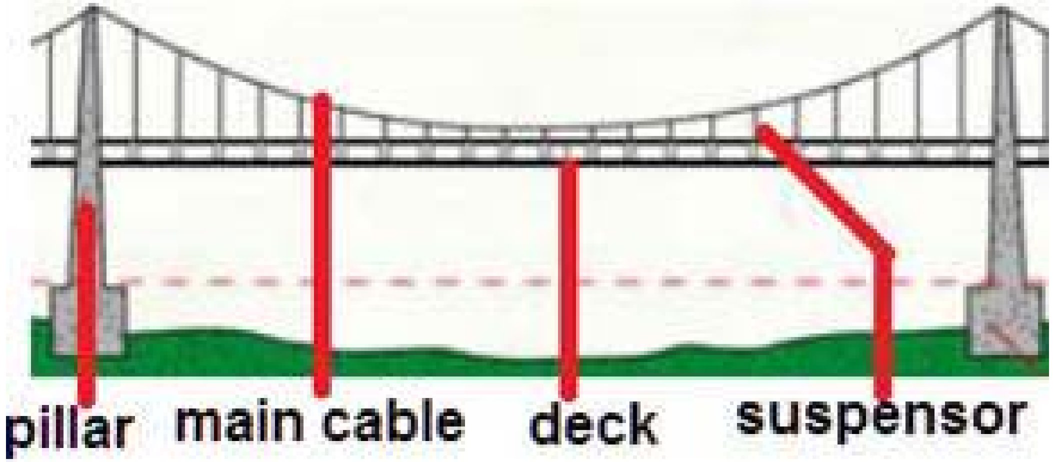
	Gr 1	Gr 2	Gr 3	Gr 4
Gr A	A1	A2	A3	A4
Gr B	B1	B2	B3	B4
Gr C	C1	C2	C3	C4
Gr D	D1	D2	D3	D4

Organization

- chosen in order that each student
  - performs by himself key tasks related to a model,
  - connects different models and associated concepts,
- consistent with the idea of several working spaces to model a complex reality

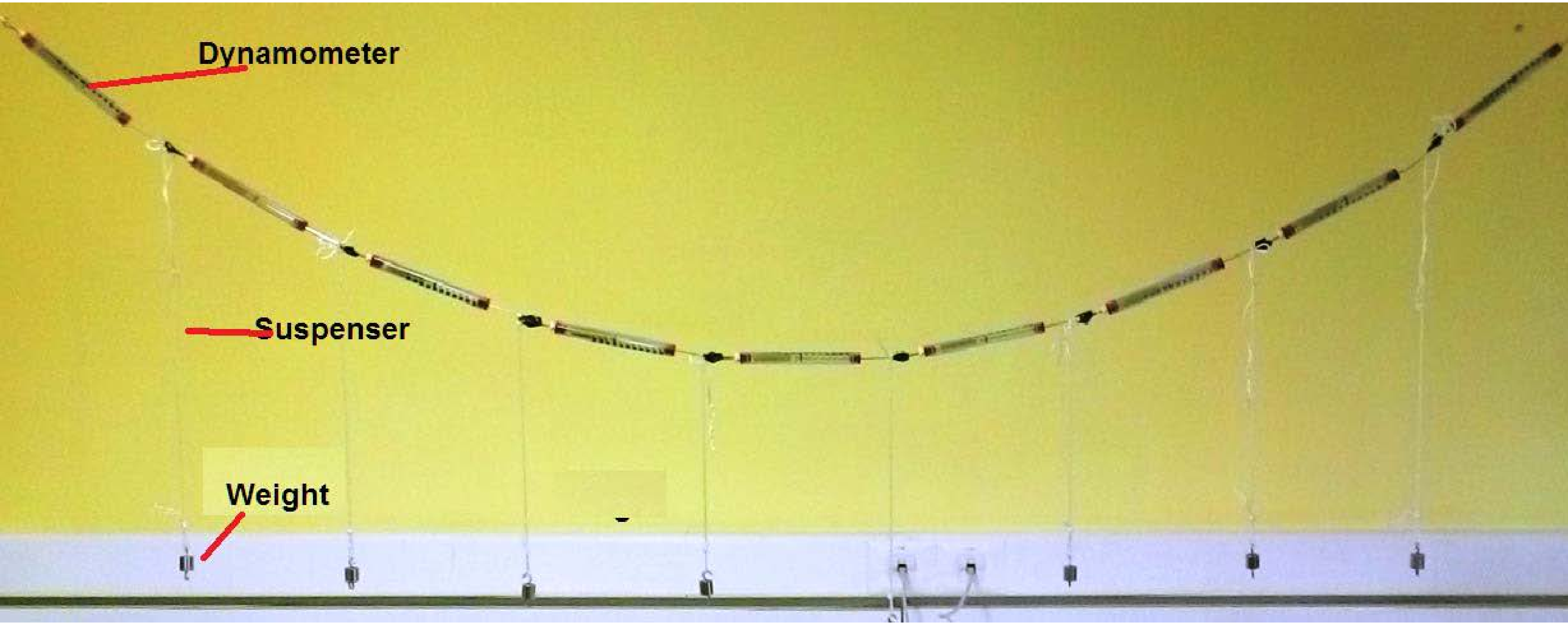
Modelling suspension bridges

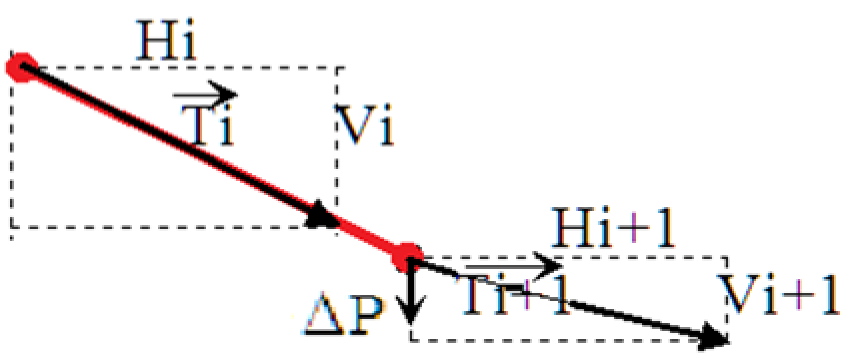
- Four models
- Four Working Spaces
- Classroom implementation (12th grade)
- Observation and evaluation
- Conclusion



- The deck is hung below main cables by vertical suspensors equally spaced.
- The weight of the deck applied via the suspensors results in a tension in the main cables.
- There is no compression in the deck and this allows a light construction and a long span (Golden Gate, Akashi kaikyō…) .
- Not to be confused with
  - Catenary (deck follows the cable)
  - Straight cables (Arena viaduct near Bilbao …)

A) Physical model of tensions





$$\begin{cases} H_{i+1} - H_i = 0 \\ V_{i+1} - V_i - \Delta P = 0 \end{cases}$$

B) Model in coordinate geometry

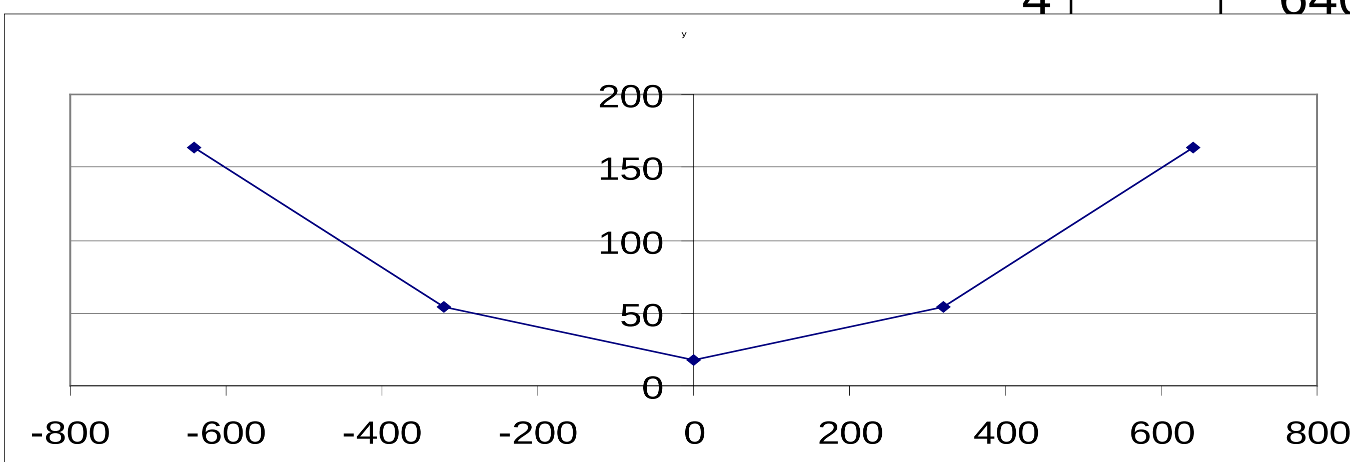
M_0 and M_n the anchoring points on the pillars, and M_1, M_2,…, M_{n-1}, the points where suspensors are attached on the cable, x_i, y_i the coordinates of M_i. The sequence of slopes (c_i) of the segment [M_i, M_{i+1}] is in arithmetic progression.

$$x_0 = -640 \quad y_0 = 163$$

$$x_1 = x_0 + \frac{1280}{n} \quad y_1 = y_0 + \frac{1280}{n} \cdot c_0$$

……………………………………………

i	ci	xi	yi
0	-0,3	-640	163
1	-0,1	-320	53,91
2	0,1	0	17,55
3	0,3	320	53,91
4		640	163



C) Algorithmic model

In the program below, the data comes from the golden gate bridge and the origin of the coordinate system is at the middle of the deck.
Weight of the deck: 20 MegaNewtons
Distance between two pillars: 1 280m
Elevation of pillars above the deck: 163m

//coordinates of the anchoring point

$x \leftarrow -640$

$y \leftarrow 163$

//Tension on the anchoring point

$\mathbf{V} \leftarrow -5$

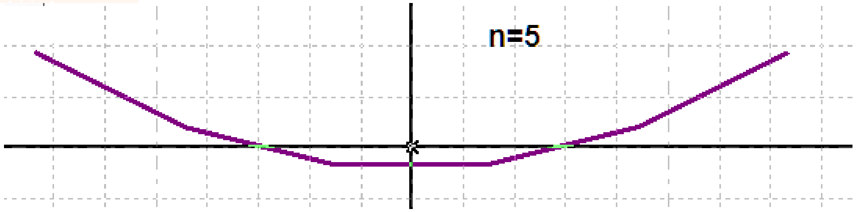
Pour i allant de 1 à n

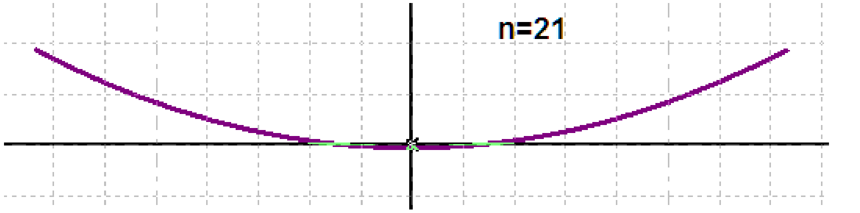
$x \leftarrow x + \frac{1280}{n}$

$y \leftarrow y + \frac{1280}{n} \cdot \left(\frac{V}{H}\right)$

$\mathbf{V} \leftarrow \mathbf{V} + \frac{10}{n-1}$

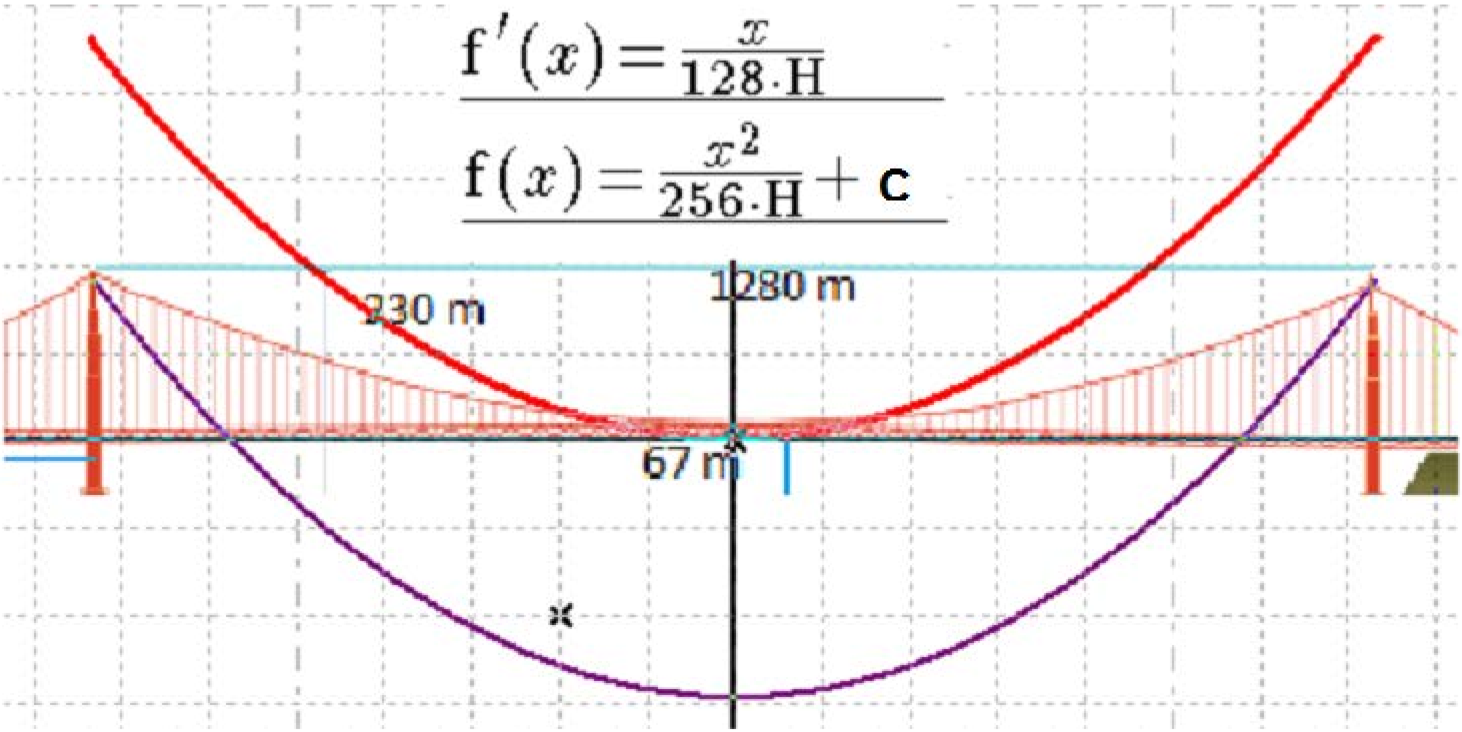
Fin Pour





D) Continuous model, using a mathematical function

V(x)= P. *x* / 2L   f '(x)= V(x)/H



Tasks for the groups of experts

- Group A: physical model
  - recognize horizontal component constant, compute a recurrence formula for the vertical components.
- Group B: geometrical model
  - compute the series of *x* and *y*-coordinates of the suspension points for a small value of *n.*
- Group C: algorithmic model
  - enter and execute the algorithm, interpret the parameter *n*, and adjust the parameter *H.*
- Group D: continuous model
  - find a formula for the derivative of *f*. Find a formula for *f* and adjust the parameter *H.*

A) Static systems working space

- Semiotic: sequence of tensions
- Discursive: static equilibrium law, properties of progressions
- Instrumental: measurement with concrete devices

B) Geometrical working space

- Semiotic: sequence of points and coordinates
- Discursive: analytical definition of a segment

C) Algorithmic working space

- Semiotic: recurrence definition of sequences expressed in the programming language
- Instrumental : programming, animation of parameters

D) Mathematical functions space

- Semiotic: standard mathematical functions
- Discursive: classical rules in calculus.
- Instrumental : graphing, CAS, animation of parameters

Classroom Implementation

- Preparation (one hour)
- Groups of experts (50 mn)
- Groups of discussion (50 mn))
- Whole class synthesis (30 mn)

Connections

Model in coordinate geometry ⟷ Algorithmic model

Evolution of the variables *x* and *y*

$$x_0 = -640 \quad y_0 = 163$$

$$x_1 = x_0 + \frac{1280}{n} \quad y_1 = y_0 + \frac{1280}{n} \cdot c_0$$

....

$$x \leftarrow x + \frac{1280}{\mathrm{n}}$$

$$y \leftarrow y + \frac{1280}{\mathrm{n}} \cdot \left(\frac{\mathrm{V}}{\mathrm{H}}\right)$$

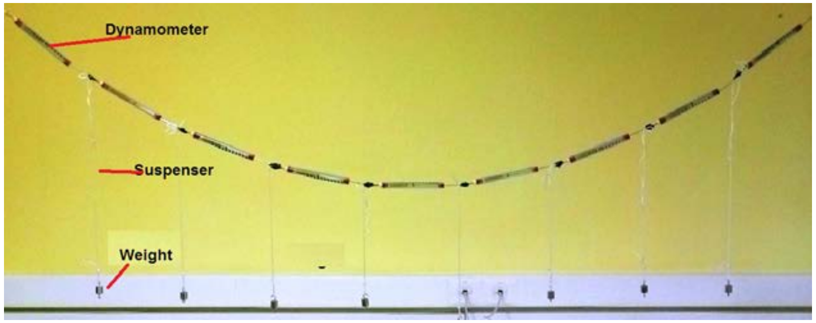
$$\mathrm{V} \leftarrow \mathrm{V} + \frac{10}{\mathrm{n}-1}$$

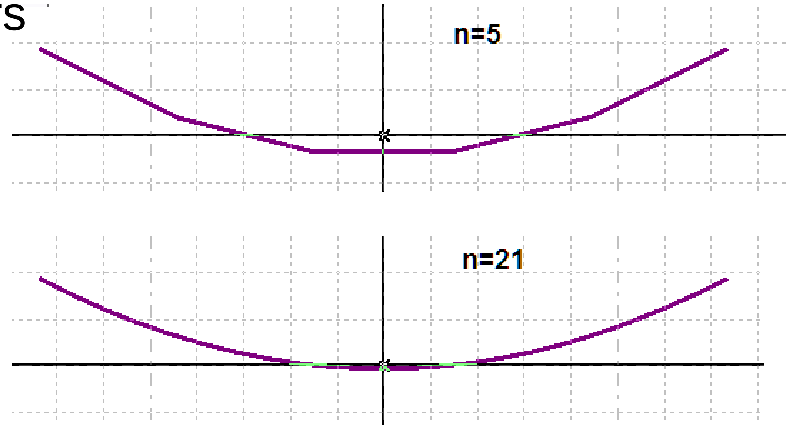
Students interpret the evolution of the variables *x* and *y* in the algorithm, by connecting the body of the loop with the recurrence law of the coordinates in the geometrical model

Connections

Physical model of tensions ⟷ Algorithmic model

Animation of parameters





Students recognize *n* as the number of suspensors and *H* as the horizontal tension

Connections

Physical model of tensions ⟷ Continuous model

Gradient in a point of the curve

$\Delta yi / \Delta xi = V_i / H_i$

$f\,'(x) = V(x) / H$

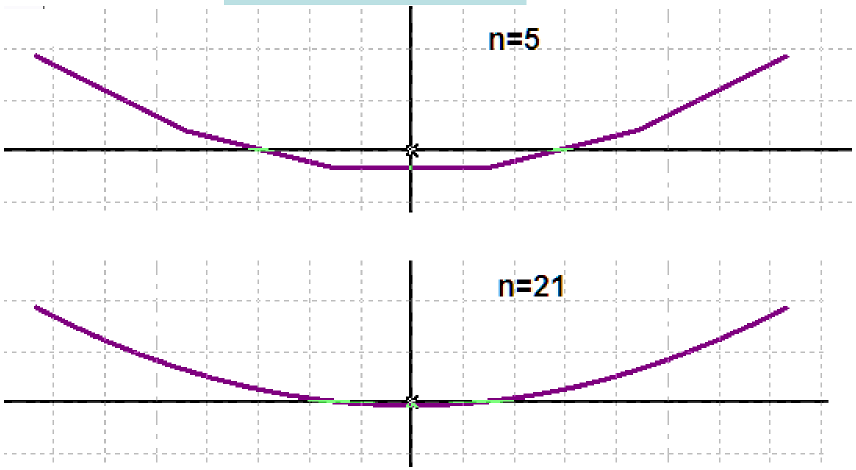
Observer asked to explain why the gradient in a point of the curve is the quotient of *V* and *H*.
Students simply wrote $f\,'(x) = \Delta y / \Delta x = V(x) / H$.

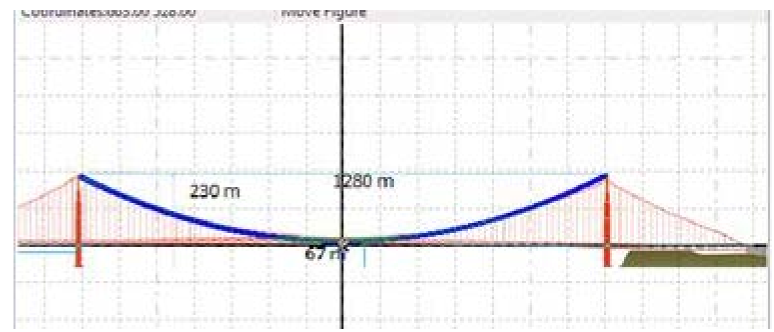
Connections

Algorithmic model

Identification of curves

Continuous model

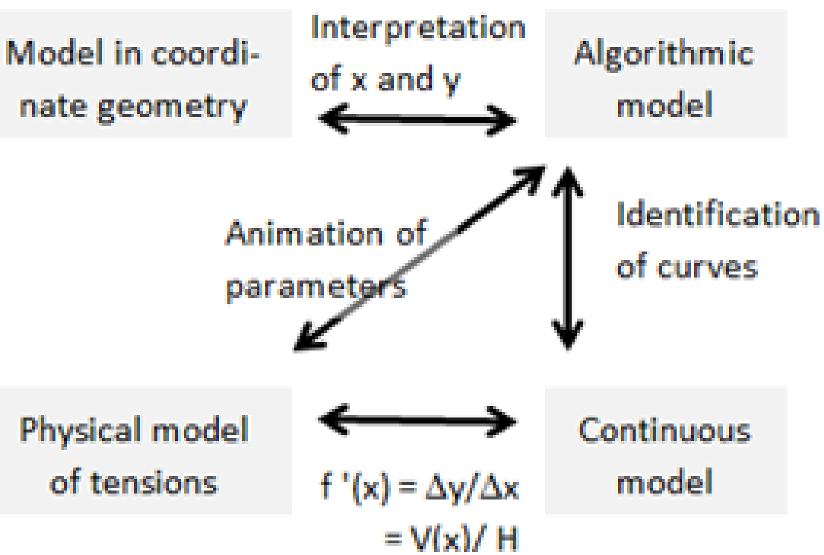


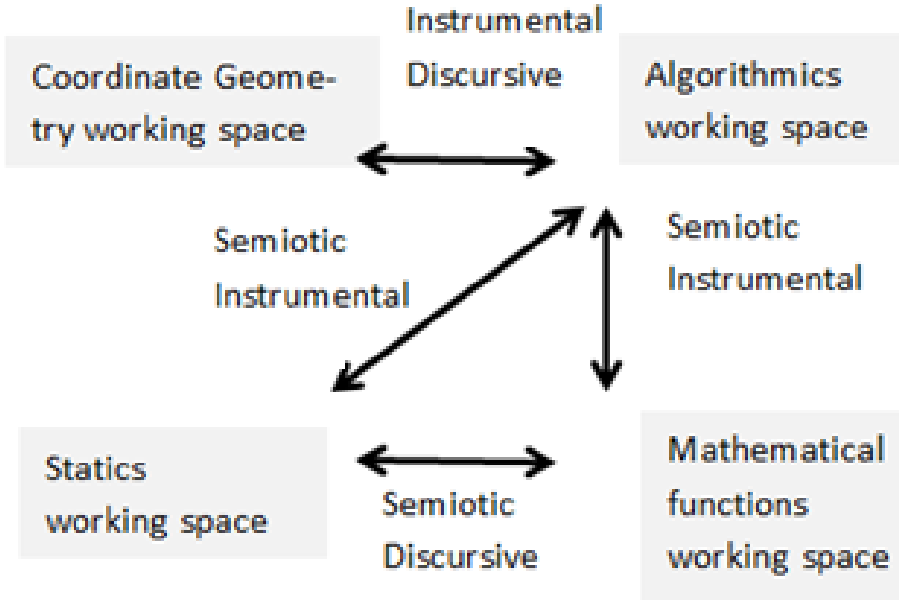


No show clear awareness that the function is the limit of the continuous piecewise function.

From graphical evidence students thought that it was more or less the same function for big values of *n*.

Connections





Conclusion

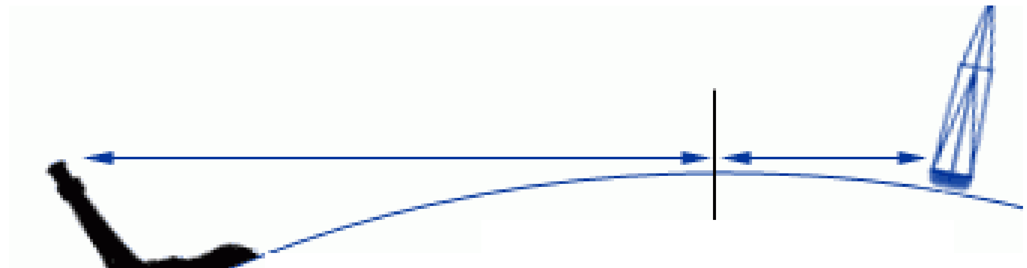
- Potential of the situation
  - Students understood the main aspects of the models and the connections between them.
  - Students understood more comprehensively concepts
    - in physics,
    - In geometry and calculus,
    - in algorithmics

  thanks to the connections.

- Potential of the framework
  - Specification of adequate working spaces
  - Adequate classroom organisation
  - Evaluation of students' modelling activity
  - Adequate integration of digital technologies

Different scientific fields
Different models of reality

A “navigational science” model:



$$d \simeq 2.08(\sqrt{h}+\sqrt{H})$$

A “geometrical-algebraic” model:

$$d = \sqrt{h^2+2\cdot R\cdot h}+\sqrt{H^2+2\cdot R\cdot H}$$

An “analytical model”:

$$d \simeq \sqrt{2R}(\sqrt{h}+\sqrt{H}) \simeq 1.93(\sqrt{h}+\sqrt{H})$$

Different Models and Working Spaces

Navigational science model

- Observation
- Table
- Practical calculation and accuracy

Geometrical-algebraic model

- Section of the earth as a “great circle”
- Pythagorean theorem
- Algebraic calculation

Analytical model

- Mathematical approximation
- Preponderence R>>h

The second phase

50 mn long, Students split into groups, each with a task

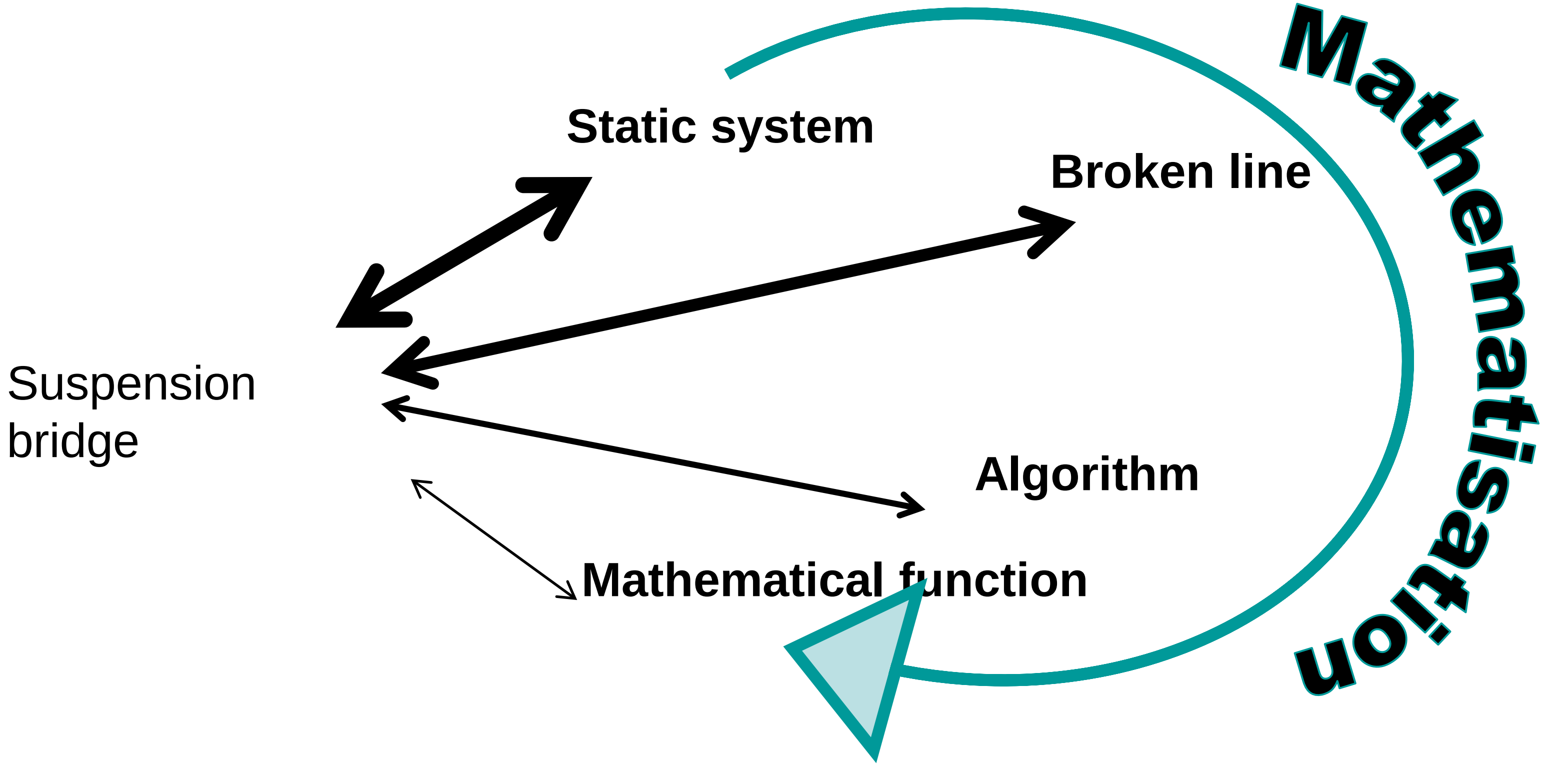
- Task A (static systems working space)
  Students have to study the sequence of horizontal and vertical components of tensions at the suspension points

- Task B (geometrical working space).
  Students have to compute the series of *x* and *y*-coordinates of the suspension points for a small number of suspensors.

- Task C (algorithmic working space).
  An algorithm given; they have to enter and execute the algorithm, interpret parameter n, and adjust parameter H

- Task D (mathematical functions working space).
  They have to search for a function *f* whose curve models the cable, find a formula for the derivative of f, then for f and adjust H

All models are mathematical, some are more



Modelling at upper secondary level

- Modelling a real life situation implies interrelated concepts
  - in physics or natural sciences,
  - In geometry
  - in calculus:
  - in algorithmics…
- The goal for students
  - not to "reinvent" each concept in isolation,
  - but rather to recognize how modelling involves understanding these concepts operationally and in interaction.