A short introduction to GeoGebra automated reasoning tools (ART)

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Our aims…

From

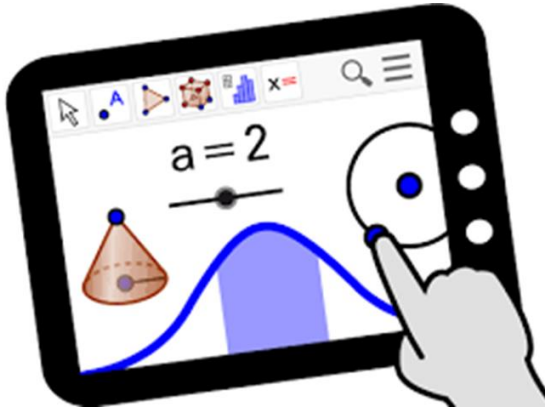
Automated reasoning tools for math learning

Use artifacts to achieve some didactic goals

to

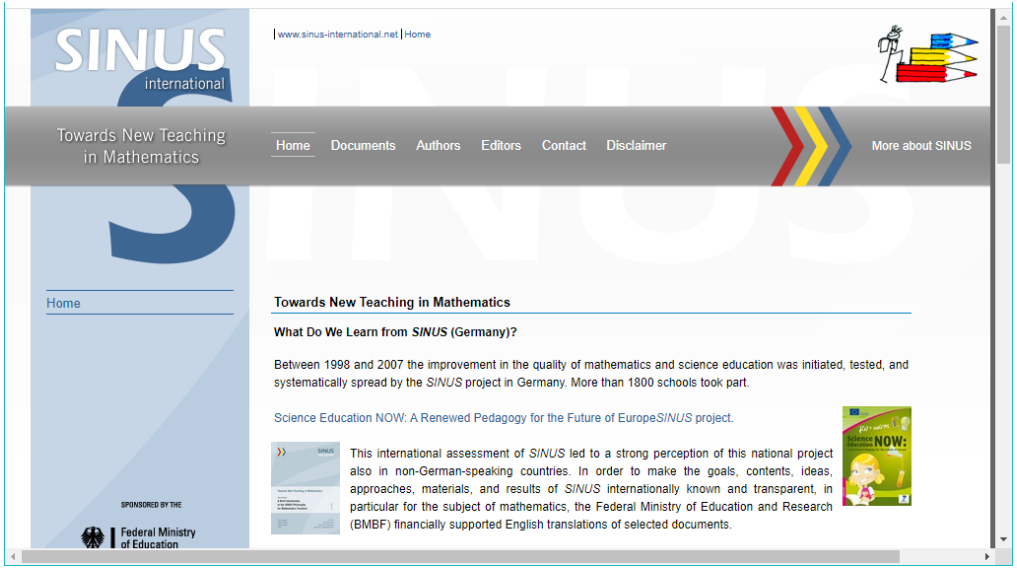
Instrumented learning for math reasoning

Reconsider didactic goals of math education according to new computer tools



Different *milieu* requires different tasks …

"**Open-ended tasks** are any tasks where students are asked to explore objects and to discover and investigate their mathematical properties" V. Ulm (The SINUS Project 1998-2007)



Volker Ulm (2011) Teaching Mathematics – Opening up Individual Paths to Learning. In: *Towards New Teaching in Mathematics,* 3, SINUS International. http://sinus.uni-bayreuth.de/math/tnt_math_03.pdf

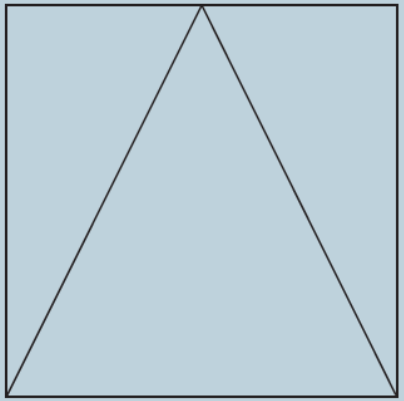
Open-ended tasks invite to explore and discover…

"the following example is another rather common-or-garden geometrical figure that can serve as a starter for mathematical exploration and discovery. This idea was developed in the context of the SINUS program implemented at the Jacob-Grimm-Schule, Rotenburg, Germany."

Triangle inscribed in a square

Here you see a square with a triangle "fitted" into it.

1. Make at least five mathematical statements pertaining to the figure (e.g. area content, angles, ...).
2. Drag the upper corner of the triangle so as to obtain an equilateral triangle. What is the height of this new triangle? What percentage of the square is taken up by that triangle?
3. How can you drag the upper corner of the triangle so as to obtain a triangle taking up a quarter of the area of the square?



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What ART is?

Recently, the computer algebra system Giac was embedded in GeoGebra (Kovács and Parisse, 2015), allowing for the implementation of automated proving algorithms based on the algebraic approach described in Recio and Vélez (1999).
The result is a collection of GeoGebra tools and commands that allow to conjecture, discover and prove statements on a given geometric construction.

- ART features available in GeoGebra 5
- New ART improvements and features in GeoGebra-Discovery Beta version
  http://autgeo.online/geogebra-discovery/

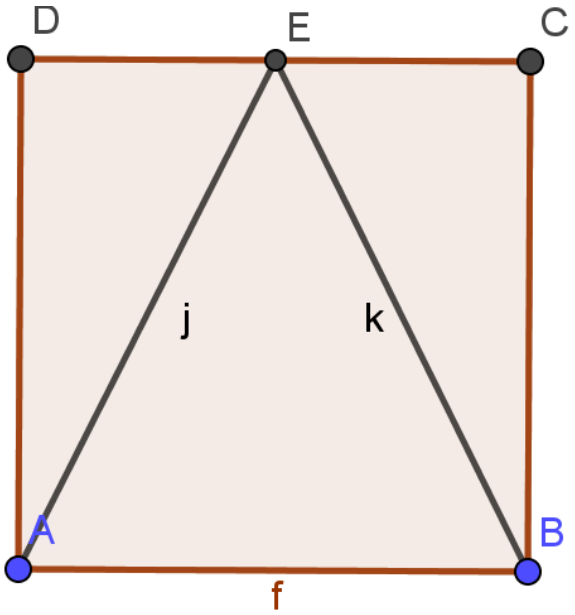
  Last release (Feb 28 2020)
  https://github.com/kovzol/geogebra/releases/tag/v5.0.560.0-2020Feb28

GeoGebra ARTools

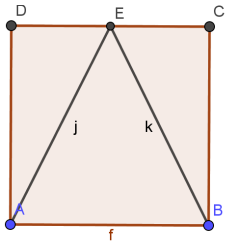
- **Relation**: automatic finding of geometric conjectures and the verification or denial of these conjectures (improved and new features).
- **LocusEquation**: calculates the implicit equation of a free point such that a given property holds (improved).
- **Prove** and **ProveDetails**: decide if a statement is true in general and, eventually, give some additional conditions for its truth (improved).
- **Envelope**: computes the equation of a curve which is tangent to a family of objects while a certain parent of the family moves on a path (improved).
- **Discover**: gives statements holding true involving one element selected by the user in the figure (new).

Conjecturing…

A common mistake: **Triangle AEB is equilateral**

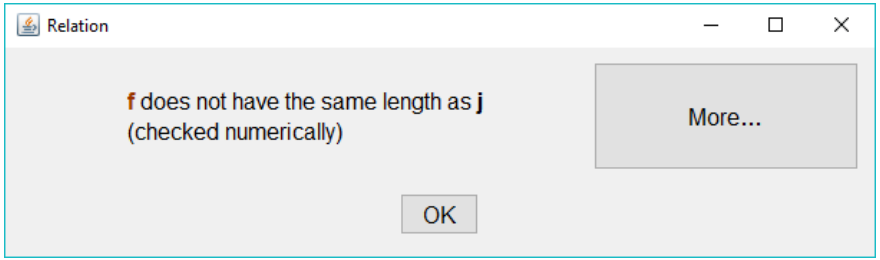


Ask GeoGebra about the relation between sides f and j or sides j and k

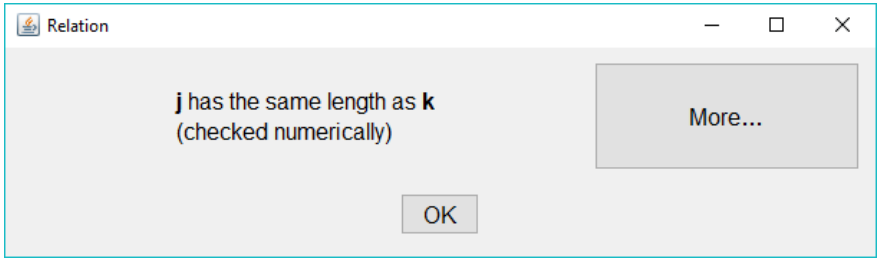


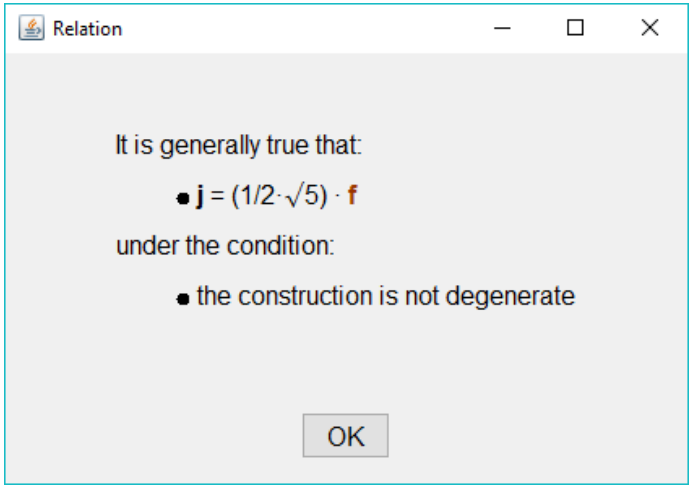
… verifying

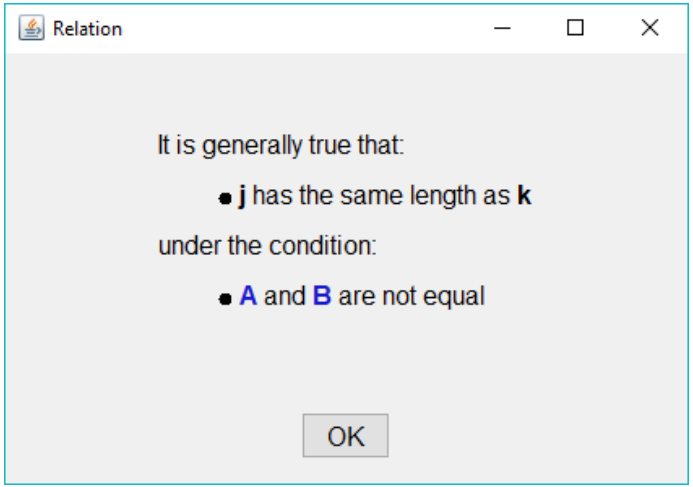
`Relation(f,j)`



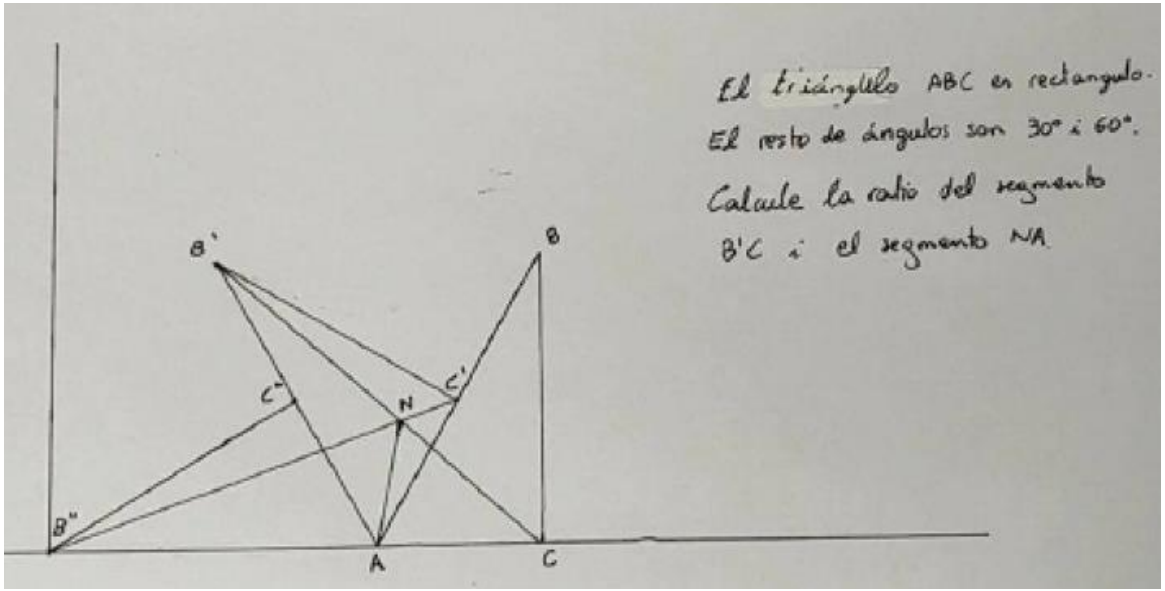
`Relation(j,k)`





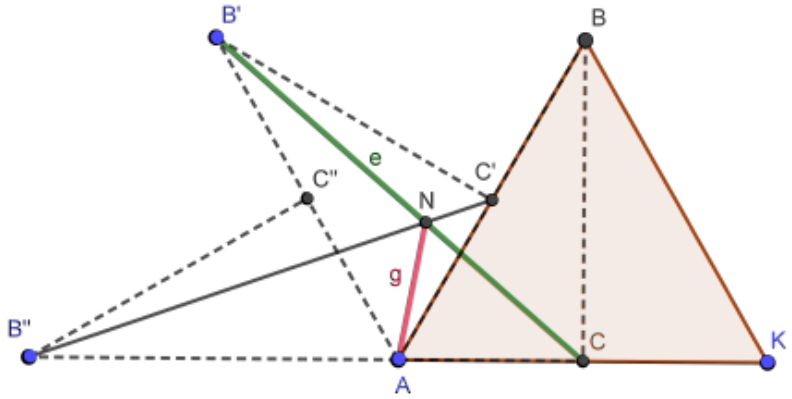


Open-ended task in examinations to become a civil servant math secondary school teacher in Spain



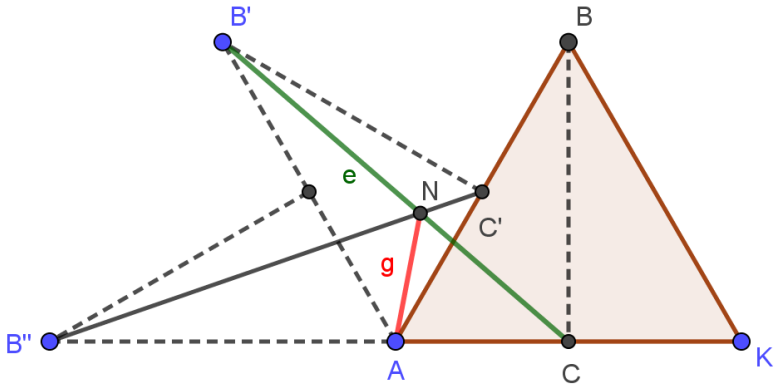
Triangle ABC is right-angled. The rest of the angles are 30 and 60º. Find the ratio between segment B'C and NA

How to draw it in GeoGebra?

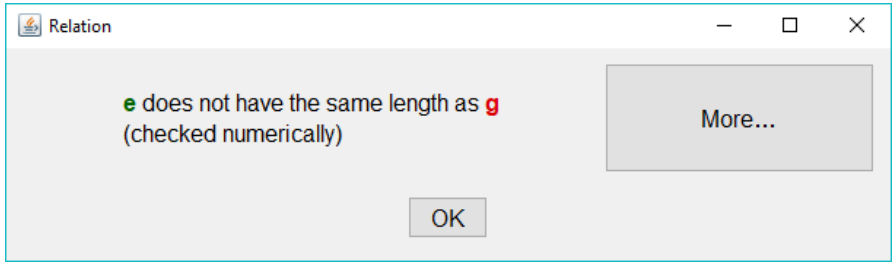


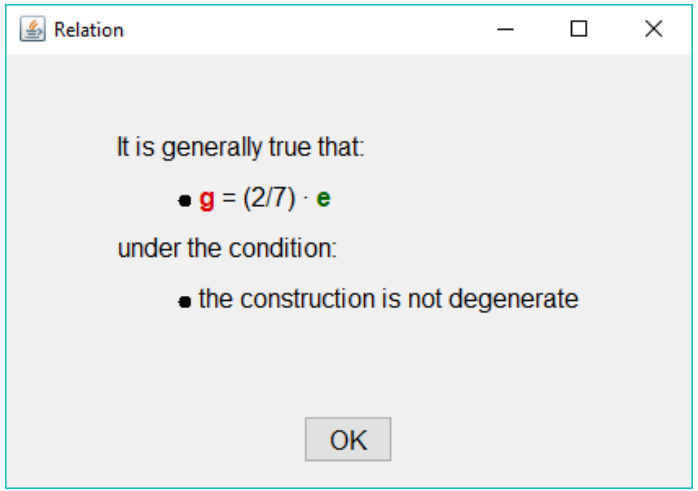
1. ABK regular triangle and C midpoint AK. The first right triangle is ABC.
2. C' midpoint AB and B' reflected point of K by C'. The second right triangle is $AB'C'$.
3. C'' midpoint of AB' and B'' the reflected point of K by A. The third right triangle is $AB''C''$.
4. N intersection point of $B'C$ and $B''C'$.
5. Label segment $B'C$ as g and segment NA as e.

Discovery using “Relation”



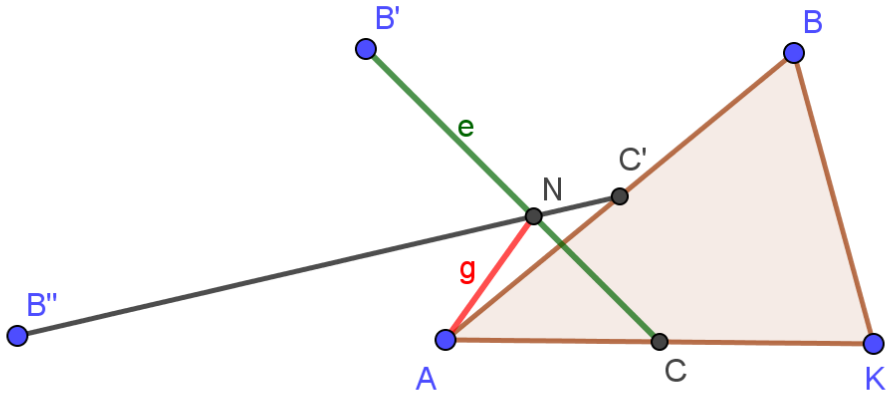
Ask GeoGebra for the ratio between e and g: **`Relation(e,g)`**



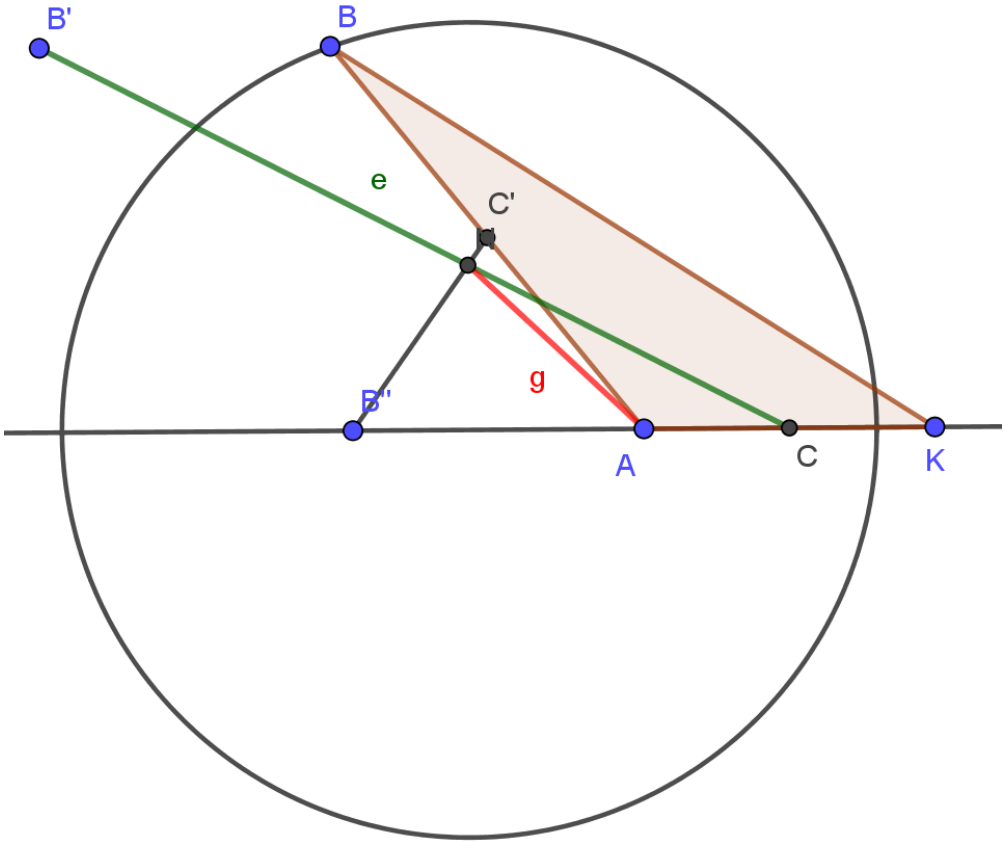


Conjecturing and discovery of new theorems

If we start with a general triangle,
how should this triangle be so that the ratio between g and e is 2/7 ≈ 0.2857?



Discovery using `LocusEquation`



The theorem is true for many non equilateral triangles

Conclusions

- **ART is a powerful tool to deal with open-ended problems**
- **ART complements a "manifold-menu" for reasoning assisted by GeoGebra:**
  - Visual: draw and move in the geometric view of GGB.
  - Numerical: check for dimensions, coordinates, equations, etc. in the algebraic view of GGB.
  - Algebraic: operate with polynomial expressions (implicit or parametric) using the CAS of GGB.
  - Automated: rigorous deductive reasoning using GGB-ART.
- **ART leads us to new geometric challenges**

To confront (and to achieve* collaboration *between) "teaching differently" with "teaching something different"!

Kovács, Z., Recio, T., Vélez, M.P. (2017) Diseño de experiencias de aula usando razonamiento automático con GeoGebra, Actas VIII CIBEM.