# Towards a mechanical geometer

Tomás Recio,
Universidad de Cantabria

http://www.recio.tk

# Abstract:

I will report on recent work on the automated discovery of properties in elementary geometry. A prototype system is being currently implemented on top of GeoGebra. The method for discovering new results from a given construction is symbolic, providing them with mathematical rigor and certainty.

After describing some generalities about the “mechanical geometer” I will end reflecting on some of the many challenges and applications it poses, in the educational system and beyond.

# Team

- Francisco Botana
- Zoltán Kovács
- Philippe R. Richard
- M. Pilar Vélez
- Steven van Vaerenbergh

Et al. : Abánades, Solyom-Gecse, Vajda, Sendra, Villarino, Martínez-Sevilla…

Howson, G. and Wilson, B. (1986):
ICMI Study series: School mathematics in the 1990's.
Kuwait: Cambridge University Press.

"…even if the students will not have to deal with computers till they leave school, it will be necessary to rethink the curriculum, because of the changes in interests that computer have brought"

Consider, for example, the following question (to other aspects of which we shall wish to refer later):

Two lines are drawn from one vertex of a square to the midpoints of the two non-adjacent sides. They divide the diagonal into three segments (see Figure 5.2).

(a) Are those three segments equal?

(b) Suggest several ways in which the problem can be generalised.

(c) Does your answer to (a) generalise?

(d) Can the argument you used in (a) be used in the more general cases?

(e) If your answer to (d) is 'No', can you find an argument which does generalise?

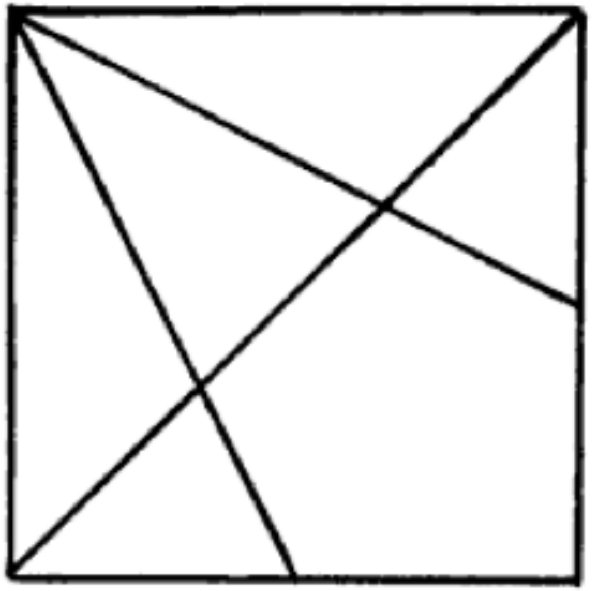
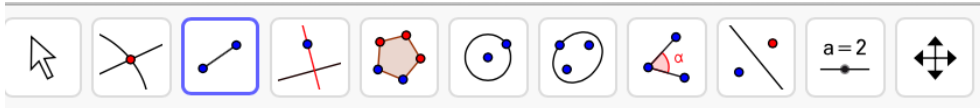


Fig. 5.2

We shall comment briefly on the purpose of such questions in Chapter 6. Here we note only that whereas the student of the 1950s had only purely geometrical ways of tackling the problem, today's student may well be able to apply algebraic methods to find the solution for (a). The solution derived by applying a mechanical procedure may be less aesthetically satisfying than a geometrical one, but are there other objections to algebraic methods than that of aesthetics? Arguments against the use of algebraic methods for the solution of geometric problems have a long history: Simson in the eighteenth century, we are told, regarded such methods as little better than a 'mechanical knack' in which the student proceeded 'without ideas of any kind' in order to 'obtain a result without meaning and without being conscious of any process of reasoning'. Similar objections were used by those supporting Euclid's geometry in the early nineteenth century, for example, Ohm in Germany and Whewell in England. They were more recently repeated by Thom (1973) who pointed out how algebra is rich in 'syntax', but weak in 'meaning', whereas geometry is the reverse.




$\ell$ = Segmento(D, E)

→ 4.96

G = Interseca(j, $\ell$)

→ (3.72, 0.05)

H = Interseca(j, k)

→ (5, 1.71)

m = Segmento(A, G)

→ 2.09

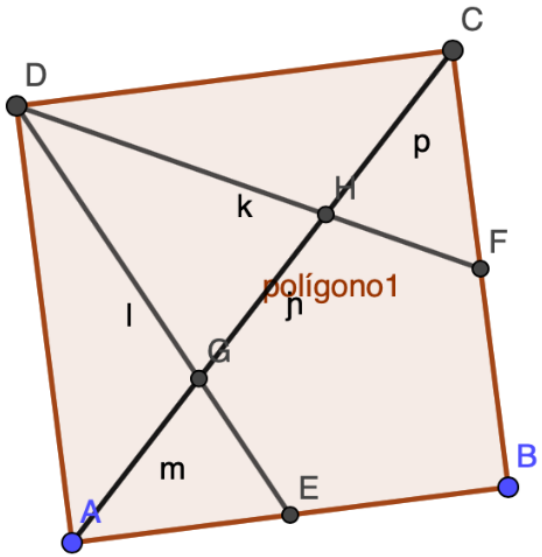
n = Segmento(G, H)

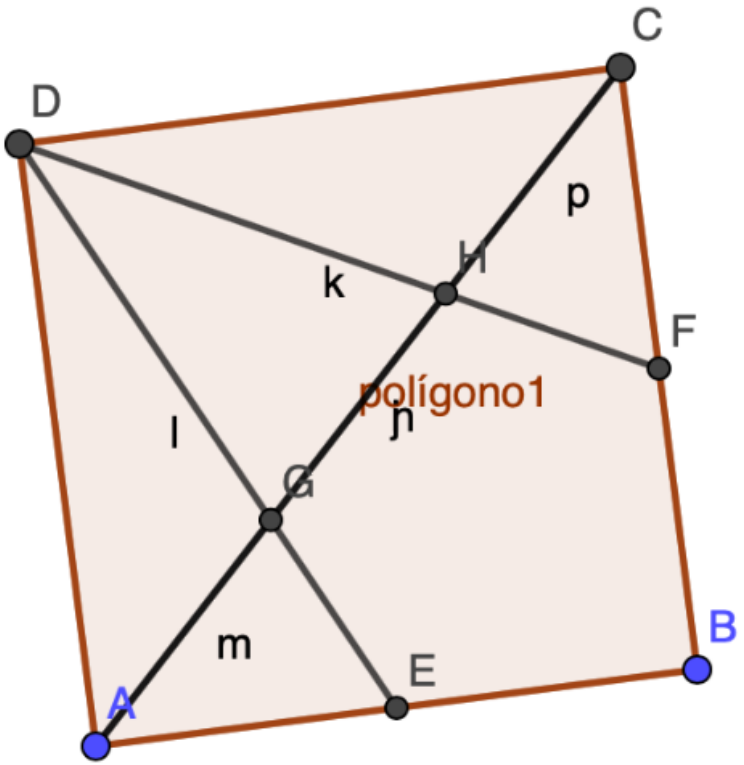
→ 2.09

p = Segmento(H, C)

→ 2.09

Relación(m, n)



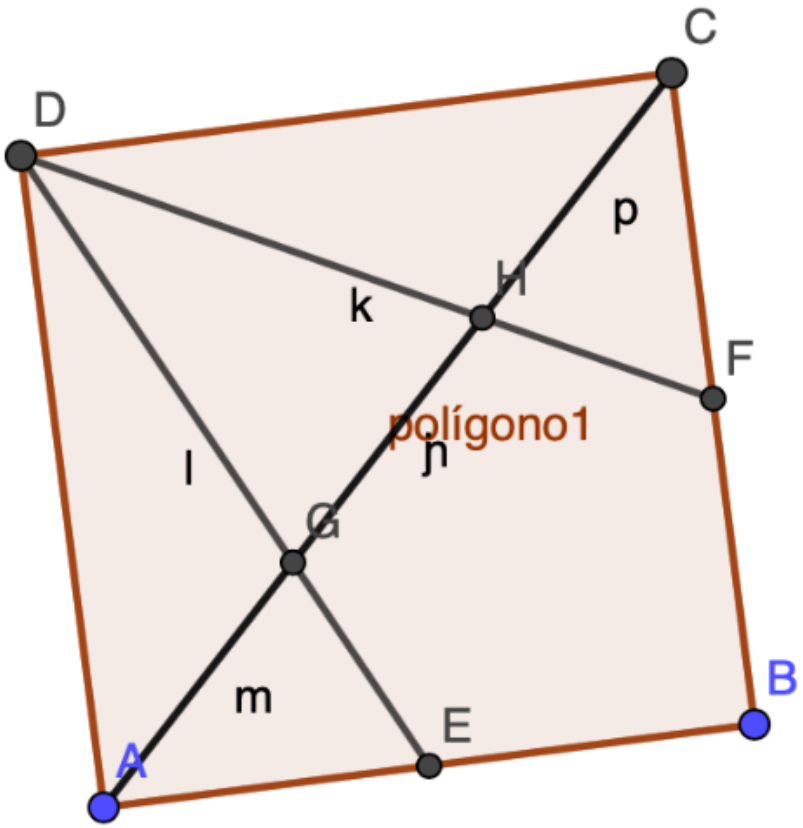



## Relación

m tiene igual longitud que n
(comprobado numéricamente) Más...

m y n son paralelas
(comprobado numéricamente) Más...

OK




## Relación

Lo que es generalmente cierto es que:

- m tiene igual longitud que n

bajo la condición:

- A y B no son iguales

m y n son paralelas
(comprobado numéricamente) Más...

OK

k : Line(D, F)

→ 0.9x + 4.15y = 6.1

ℓ : Line(A, C)

→ -4.62x + 2.74y = -5.81

G = Intersect(j, ℓ)

→ (0.97, -0.48)

H = Intersect(k, ℓ)

→ (1.89, 1.06)

m = Segment(A, G)

→ 1.79

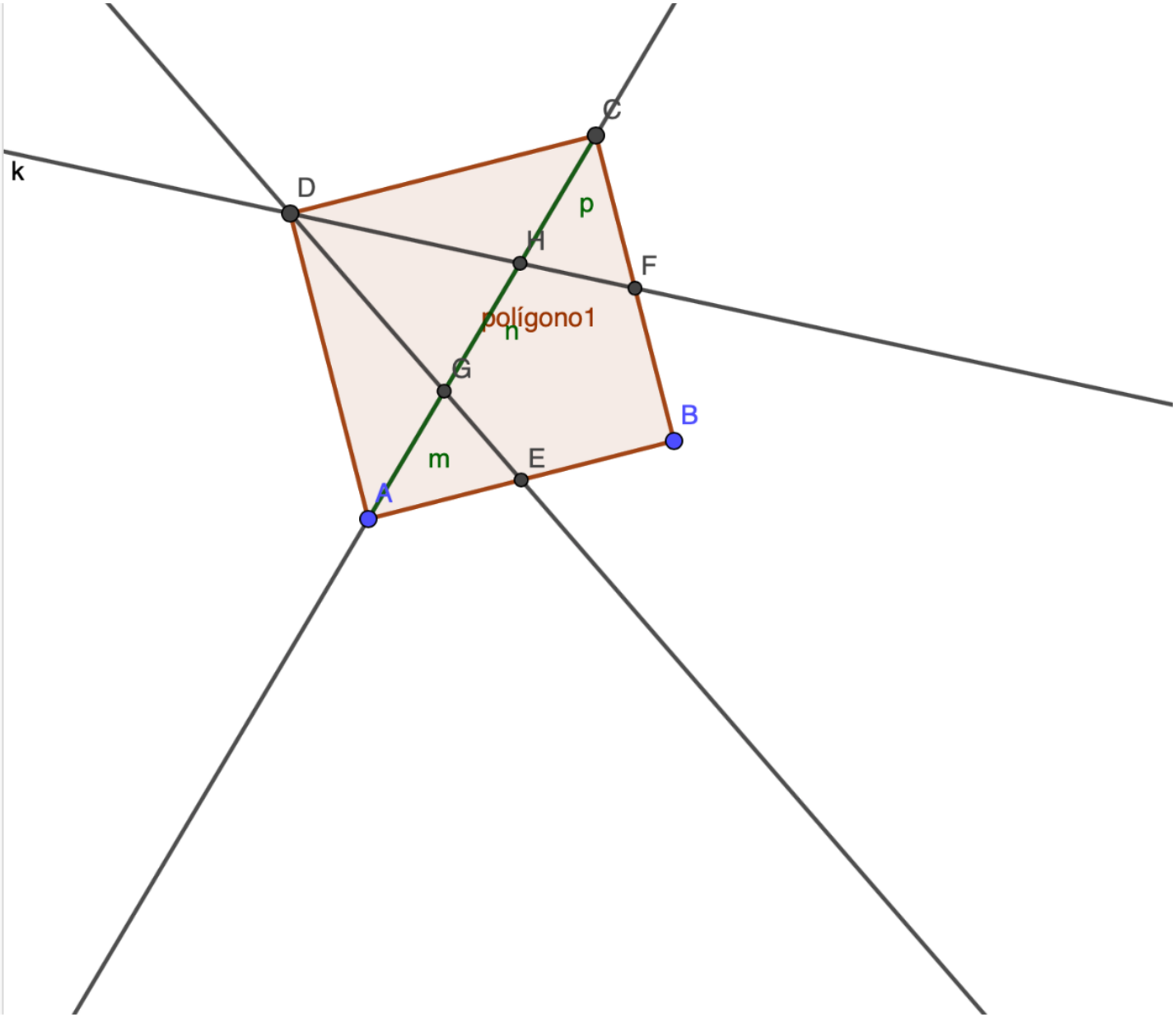
n = Segment(G, H)

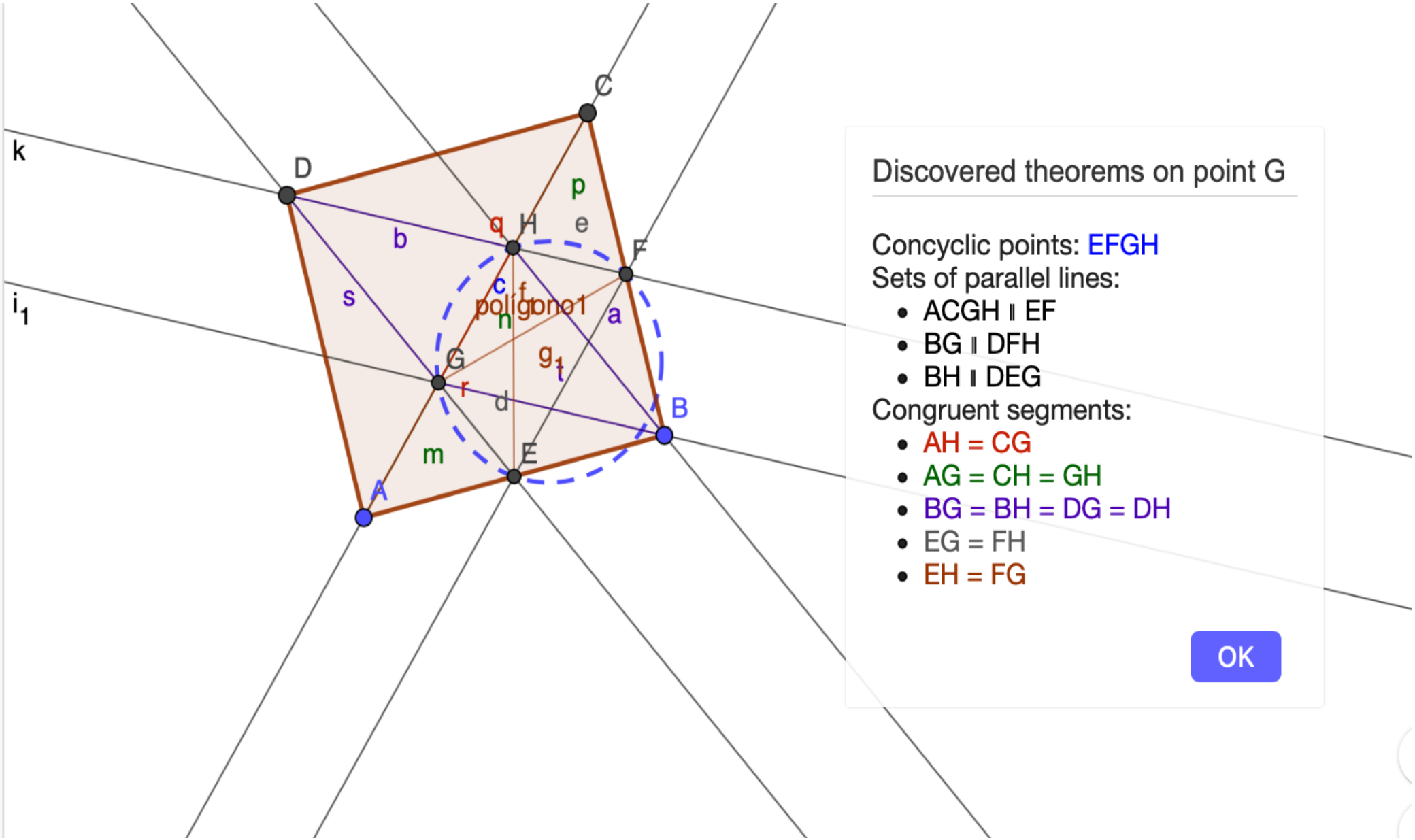
→ 1.79

p = Segment(H, C)

→ 1.79

Discover(G)






## Discovered theorems on point G

Concyclic points: EFGH
Sets of parallel lines:

- ACGH ∥ EF
- BG ∥ DFH
- BH ∥ DEG

Congruent segments:

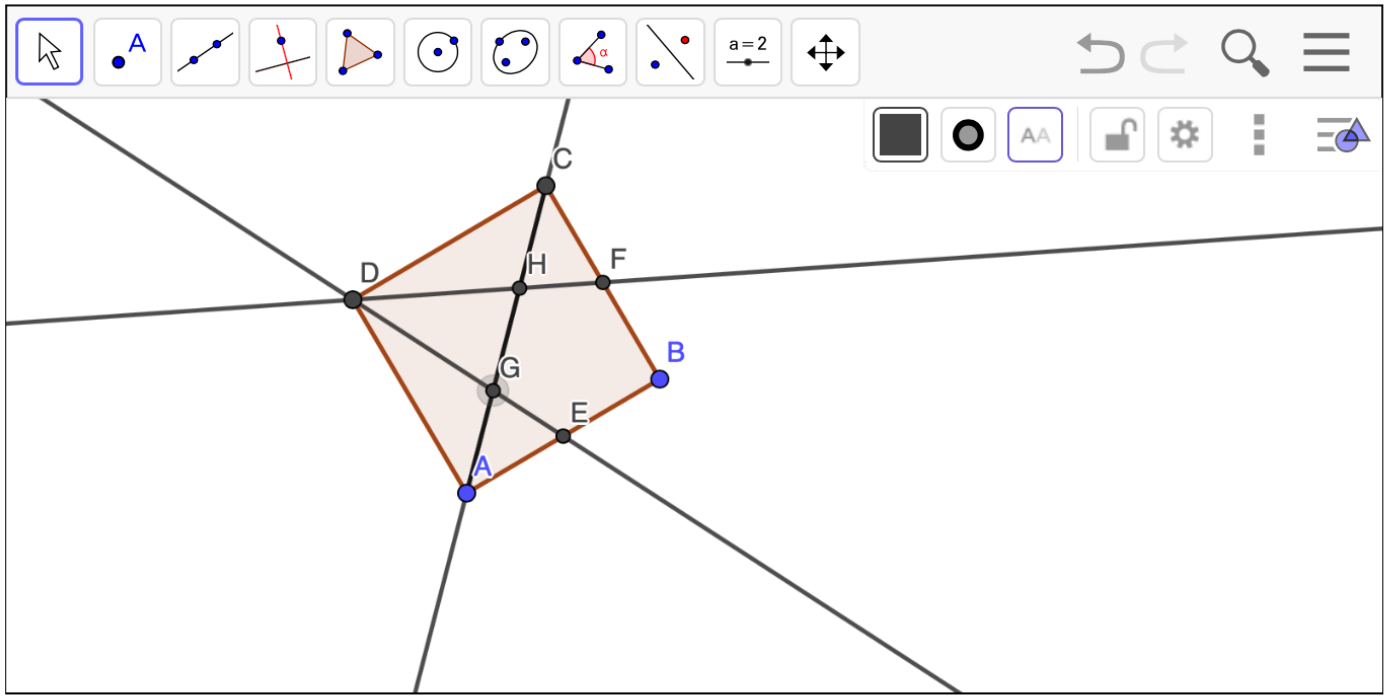
- AH = CG
- AG = CH = GH
- BG = BH = DG = DH
- EG = FH
- EH = FG

OK

# Welcome to the Automated Geometer!

Using GeoGebra 5.0.495.0 (offline).

Let us consider this initial input construction (you may freely edit the construction or upload one as well; only the visible points will be observed; also you can load ar
Thales' circle theorem (or a variant), regular hexagon, 9 points circle, 7 circles; or start something new) :




Select relations to check:

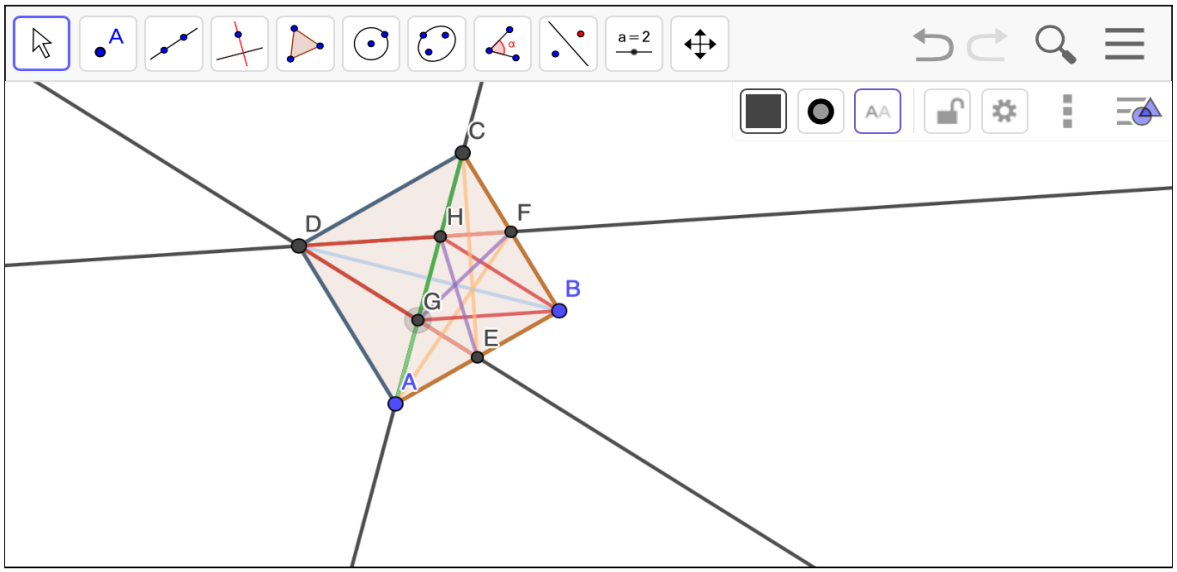
- Collinearity of three points
- Equality of distances between two points
- Perpendicularity of segments defined by two points
- Parallelism of segments defined by two points
- Concyclicity of four points

Start discovery

# Welcome to the Automated Geometer!

Using GeoGebra 5.0.495.0 (offline).

Let us consider this initial input construction (only the visible points will be observed) :




Select relations to check:

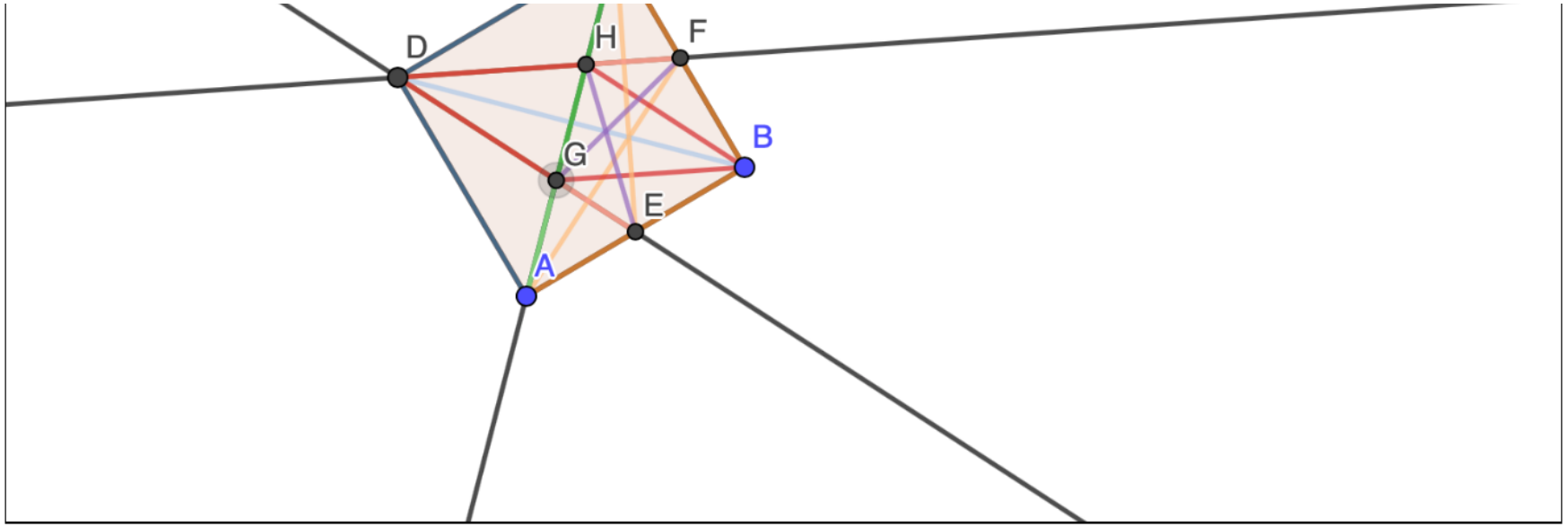
Collinearity of three points
Equality of distances between two points
Perpendicularity of segments defined by two points
Parallelism of segments defined by two points
Concyclicity of four points

The following theorems can be proven:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. AB=AD | 5. AD=BC | 9. AE=CF | 13. AG=CH | 17. BE=BF | 21. BG=DG | 25. CE=DE | 29. DG=DH |
| 2. AB=BC | 6. AD=CD | 10. AF=CE | 14. AG=GH | 18. BE=CF | 22. BG=DH | 26. CE=DF | 30. EG=FH |
| 3. AB=CD | 7. AE=BE | 11. AF=DE | 15. AH=CG | 19. BF=CF | 23. BH=DG | 27. CH=GH | 31. EH=FG |
| 4. AC=BD | 8. AE=BF | 12. AF=DF | 16. BC=CD | 20. BG=BH | 24. BH=DH | 28. DE=DF | |

Finished, found 31 theorems among 378 possible statements.
Elapsed time: 0h 0m 2s

Restart with a new or the same experiment




Select relations to check:

Collinearity of three points
Equality of distances between two points
Perpendicularity of segments defined by two points
Parallelism of segments defined by two points
Concyclicity of four points

The following theorems can be proven:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. AB=AD | 5. AD=BC | 9. AE=CF | 13. AG=CH | 17. BE=BF | 21. BG=DG | 25. CE=DE | 29. DG=DH |
| 2. AB=BC | 6. AD=CD | 10. AF=CE | 14. AG=GH | 18. BE=CF | 22. BG=DH | 26. CE=DF | 30. EG=FH |
| 3. AB=CD | 7. AE=BE | 11. AF=DE | 15. AH=CG | 19. BF=CF | 23. BH=DG | 27. CH=GH | 31. EH=FG |
| 4. AC=BD | 8. AE=BF | 12. AF=DF | 16. BC=CD | 20. BG=BH | 24. BH=DH | 28. DE=DF | |

Finished, found 31 theorems among 378 possible statements.
Elapsed time: 0h 0m 2s

Algorithms, Combinatorics?

vs

AI, Machine Learning?

- Geometric Calculator…
- Interestingness, Geometrography, Complexity,….

# Didactical Issues: the role of reasoning and proving

As an auxiliary tool, what opportunities, what differences involve using GeoGebra ART?

- Can guide student exploration, provide hints, answer (partially) questions…
- Can help building up diagrams, locus…
- Helping teachers!

- Math proofs: an identity feature.

- …but, what if the output of reasoning can be automatically obtained?

- We can teach neither **as** nor **the** mathematics of the past….

- Not, particularly, in the case of Euclidean geometry, which is so much **tool driven**

- The message is clear: classical proof must move over…

**The Rise, Fall, and Possible Transfiguration of Triangle Geometry: A Mini-History,** Philip J. Davis. *The American Mathematical Monthly*, Vol. 102, No. 3. (Mar., 1995), pp. 204-214.

>>> From doing things better to doing better things?

- Gelernter, H. (1959): *Realization of a geometry theorem proving machine.* Proc. Int. Conf. on Information Processing. Paris, UNESCO house. pp. 273-282.

- QED Manifesto (1994): “QED is … a project to build a computer system that effectively represents all important mathematical knowledge and techniques. ”
  https://www.cs.ru.nl/~freek/qed/qed.html,
  https://en.wikipedia.org/wiki/QED_manifesto

- https://www.researchgate.net/profile/Tomas_Recio


- X.Chen, D. Wang: Towards an electronic geometry textbook. In: Automated Deduction in Geometry (F. Botana and T. Recio, eds.), Lecture Notes in Artificial Intelligence 4869, pp. 1–23. Springer, Berlin Heidelberg, 2007.
- P. R. Richard; J.M. Fortuny,; M. Hohenwarter, M. Gagnon**:** geogebraTUTOR : une nouvelle approche pour la recherche sur l'apprentissage compétentiel et instrumenté de la géométrie à l'école secondaire. In T. Bastiaens & S. Carliner (Eds.), *Proceedings of E-Learn 2007--World Conference on E-Learning in Corporate, Government, Healthcare, and Higher Education* (pp. 428-435). Quebec City, Canada: Association for the Advancement of Computing in Education (AACE).
- J. Davenport, J. Fleuriot, P. Quaresma, T. Recio, D. Wang: Intelligent Geometry Tools, Proc. Second International Workshop on Automated Reasoning: Challenges, Applications, Directions, Exemplary Achievements, Natal, Brazil, August 26, 2019, EPTCS 311,


Towards a COST Network proposal!

www.math.rutgers.edu/~zeilberg/PG/gt.html

# Plane Geometry:

# An Elementary Textbook

## By

## Shalosh B. Ekhad, XIV

## (Circa 2050)

### Downloaded from the Future

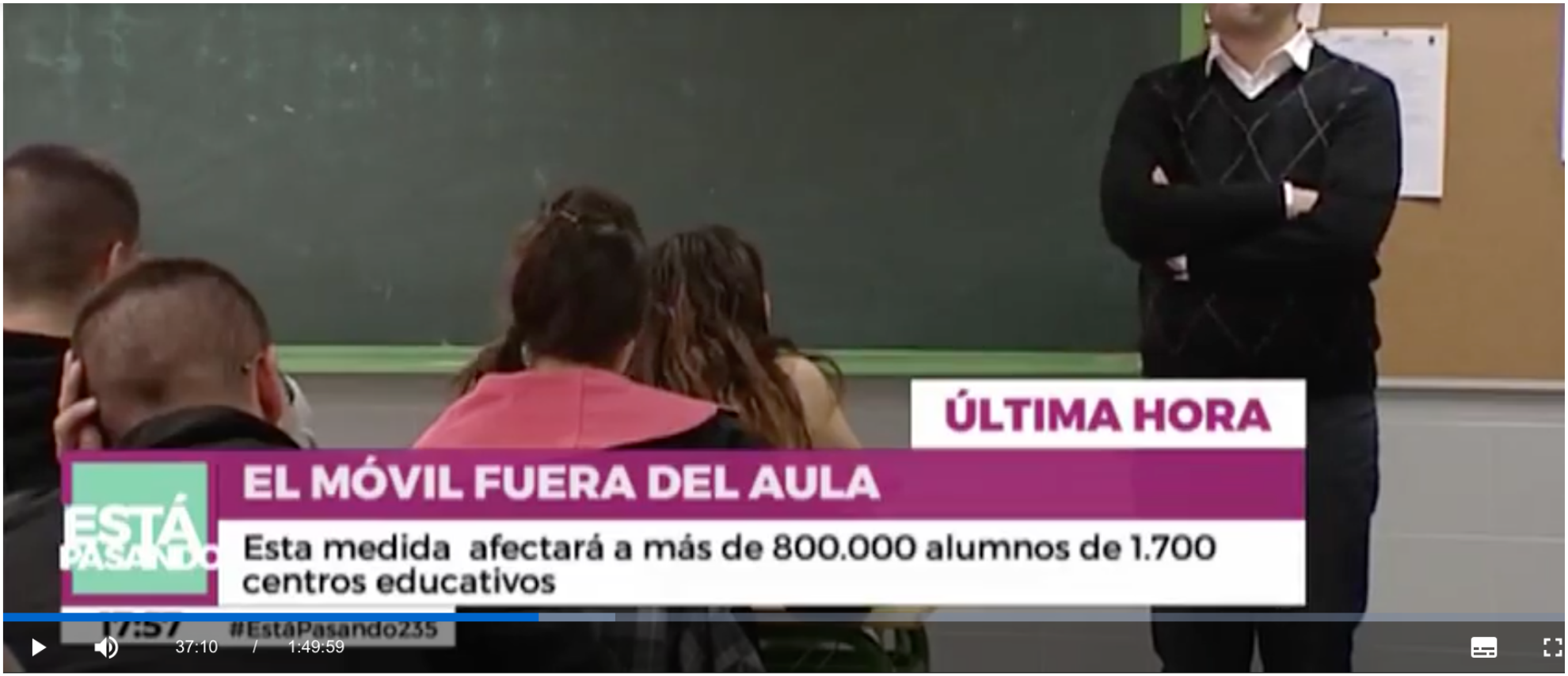
## By

## Doron Zeilberger

Foreword Introduction Definitions Theorems

RENE PictRENE

TeleMadrid




# Está Pasando 20.02.2020