TG3 Panel:
Dynamic Geometry for Mathematics Education

Eugenio Roanes-Lozano
(Chair of the TG3 Panel)

Instituto de Matemática Interdisciplinar (IMI) &
Departamento de Didáctica de las Ciencias Experimentales,
Sociales y Matemáticas,
Facultad de Educación,
Universidad Complutense de Madrid, Spain

This is a summary of the *TG3 Panel: DGS for mathematics education* that took place at the Symposium on Artificial Intelligence for Mathematics Education (AI4ME), held at CIEM Castro Urdiales, February 28th - March 1st, 2020.

Summary

The *TG3 Panel: Dynamic Geometry for Mathematics Education* focuses on the influence of the arrival of Dynamic Geometry Systems (DGS) on mathematics education. Nowadays, the evolution of the possibilities of the available DGS and the unprecedented spread of the DGS GeoGebra lead to many open questions, like:

- Do teachers (and textbooks) really take advantage of all the possibilities of new DGS?
- Should DGS affect to the way we teach? (for instance, should we teach in a more experimental way?)
- Should DGS change what we teach?
- Should DGS change how we structure what is taught?

Four exciting talks addressing applications of DGS to mathematics education took place in this panel:

1. Pilar Vélez: *A short introduction to GeoGebra automated reasoning tools (ART).*
2. Thierry Dana-Picard: *Automated exploration of envelopes.*
3. Cristina Naya: *Teaching of Geometry with GeoGebra software in students of the Primary Education Degree.*

4. Eugenio Roanes: *A constructive educational approach to conics and quadrics allowed by the arrival of 3D-DGS.*

that was followed by a discussion and final conclusions.

In my opinion, the four talks look very different, but there is a common core: the important influence of DGS in teaching mathematics and the collateral pedagogical issues that arise.

There are two eye catching issues in the present DGS development:

- The extension of 3D capabilities.
- The collaboration with Computer Algebra Systems (CAS), that opens a new world of possibilities if compared with "standard" DGS, and can be applied to all levels of mathematics education, from Secondary Education to Teacher Training and first years at universities (both Science and Engineering schools).

Let us continue with a summary of the talks and the ulterior discussions.

Pilar Vélez gave a summary of the possibilities of an impressive new tool (still under development) for automated reasoning in GeoGebra. This extension of GeoGebra opens a new field in maths teaching, as, for the first time since he beginning of mathematics teaching with technology, the students can not only explore but also obtain a confirmation of the formal truthness of a geometric result (and even been suggested new results).

Thierry Dana-Picard showed us a surprising example about how "dragging and adjusting" with a DGS could make possible for Secondary Education students to face tasks (in this case determining the envelope of a family of curves) that, without technology would be impossible to achieve at this level.

Cristina Naya gave examples of how the (rule and compass) inconstructibility and the inexistence of certain geometric configurations could be treated with a DGS at Teacher Trainining level. Curiously, she underlined the rejection of technology by some of the students, although we (teachers) consider that it is engaging for all young people.

Finally, my talk presented how a 3D DGS with algebraic capabilities could be used to introduce conics as sections of a right circular (Apollonius' cone) in a constructive way (not so algebraic and more visual). It was shown how quadrics of revolution can be also presented in a constructive way. A question arose at the end: whether these topics should be taught or not or to whom.

Some didactics ideas arose along the final discussion:

- Should geometry curricula change due to the availability of the new DGS?
- Thinking about an average Secondary School student, the importance of formal proofs possibly decreases with tools that allow to convince through checking the "stability" of a construction (and/or the answer of a black box for automatic theorem proving).
- These powerful DGS allow to explore at Secondary School level issues allocated at university level (that is, to "lower" contents).

- DGS didactic possibilities are usually compared to those of rule and compass, but they are key to treat questions such as inexistence or inconstructibility of a geometric configuration.
- Similarly, the approach of the last talk requires a (very special) DGS.
- Finally, the didactical possibilities of 3D DGS with virtual reality look huge.

To conclude, DGS are evolving very quickly lately, specially GeoGebra, after some years of a "stabilization" of their goals (let us remember that the first DGS appeared back in the early '90s). They are becoming THE TOOL for geometry teaching and are opening new possibilities and ways to teach.

I would like to thank the organizers of AI4ME, Philippe R. Richard, Steven van Vaerenbergh and Pilar Vélez for inviting me to chair this panel.